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Evaluating Urban Efficiency in Air Pollution Management: A DEA Study of 20 Iranian Cities

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Abstract

Data Envelopment Analysis (DEA) is a widely recognized quantitative tool for evaluating the efficiency of cities in managing resources and mitigating air pollution. This study considers controllable inputs, such as green space and investment in urban infrastructure, and incorporates undesirable outputs, including CO, NO_2, and PM2.5 concentrations, to reflect the negative impact of pollution. Considering these outputs allows for a more realistic and practical assessment of urban performance. The efficiency of 20 major Iranian cities is evaluated using classical DEA models with Variable Returns to Scale (VRS). Including undesirable outputs ensures that higher pollution levels reduce efficiency scores, reflecting environmental degradation. Efficiency targets for inefficient cities are computed, indicating practical adjustments in controllable inputs without expecting immediate reductions in pollution, which are influenced by multiple uncontrollable factors [1]. These results provide actionable insights for urban managers and policymakers. By comparing actual inputs with DEA-based targets, cities can benchmark performance, prioritize interventions, and adopt best practices from more efficient cities. The combination of DEA with undesirable outputs, along with managerial judgment, provides a practical framework for improving resource allocation and environmental performance in Iranian cities [2], [3].

Keywords: Data envelopment analysis, Air pollution, Undesirable outputs.

1 | Introduction

Air pollution is one of the major challenges facing large cities in Iran, with significant economic, social, and environmental consequences. Optimal management of urban resources, particularly investments in green spaces and related infrastructure, can play a crucial role in mitigating the adverse effects of air pollution. However, managerial decision-making regarding resource allocation and city performance evaluation requires quantitative, systematic tools to assess city performance across multiple indicators.

Data Envelopment Analysis (DEA) is a powerful non-parametric method for evaluating the relative efficiency of Decision-Making Units (DMUs). By using efficiency frontiers, DEA allows comparisons of multiple units

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with multiple inputs and outputs. In environmental studies related to air pollution, inputs typically include controllable resources such as green space area (ha) and investment in green space development (billion Toman). In contrast, outputs include pollution indicators such as CO, NO_2, and PM2.5 [4], [5], [6].

Matin Nejati et al. [7] presented a method based on a Genetic Algorithm to specify the most efficient unit. Moreover, Malakouti et al. [8] applied a genetic algorithm to determine the efficient frontiers of the production possibility set.

Given that outputs (pollution indicators) are influenced by multiple external factors beyond urban management's control, reducing them is not feasible in the short term. Therefore, an input-oriented approach is more appropriate for evaluating city efficiency. This approach assists managers in achieving maximum efficiency from available resources to mitigate the impact of air pollution [9].

2.2 | Theoretical Foundations of DEA

Suppose that n DMUs produce s outputs by consuming m inputs. Also, suppose that in the case where the units are black boxes, $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ represents the vector of inputs and $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$ represents the vector of outputs of the DMU_j [10]. The fractional model for calculating the relative efficiency of the DMU_o, $o \in \{1, \dots, n\}$, is as follows

$$\begin{aligned} &\theta_o = \text{Max} \sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io}, \\ &\text{s.t.} \\ &\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, j = 1, \dots, n, \\ &u_r, v_i \ge 0, r = 1, \dots, s, i = 1, \dots, m. \end{aligned} \tag{1}$$

$U = (u_1, u_2, \dots, u_s)$ and $V = (v_1, v_2, \dots, v_m)$ are the weights of the output and input vectors of DMU_o, respectively [11]. *Model (1)* for the evaluation of the DMU_o in the technology of Constant Returns to Scale (CRS) in input-oriented is known as the fractional model [12]. The above model is transformed into the following Linear Programming (LP) model, which is known as the multiplier CCR model:

$$\begin{aligned} &\beta_o = \text{Max} \sum_{r=1}^{s} u_r y_{ro}, \\ &\text{s.t.} \\ &\sum_{i=1}^{m} v_i x_{io} = 1, \\ &\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, j = 1, \dots, n, \\ &u_r, v_i \ge 0, r = 1, \dots, s, i = 1, \dots, m. \end{aligned} \tag{2}$$

The CCR multiplier model is used to calculate each DMU's efficiency score using a flexible set of weights. For each DMU, input and output weights are obtained by solving *Model (2)*. In the input-oriented approach, efficiency in DEA is defined as the ratio of the weighted sum of outputs to the weighted sum of inputs. This definition may lead to the problem of zeroing some weights, which is addressed by incorporating a non-Archimedean epsilon value [13].

3 | Methodology

DEA is recognized as a useful quantitative tool for evaluating the efficiency of cities in resource management and air pollution mitigation. This technique allows for comparing cities based on inputs and outputs, helping identify efficient and inefficient cities. However, using DEA alone is insufficient for managing air pollution, and supplementary analyses that account for the specific characteristics of urban environments are required. A key aspect in air pollution analysis is the consideration of undesirable outputs. These outputs include pollution indicators such as CO, NO_2, and PM2.5, whose reduction is influenced by multiple factors beyond urban management control. Incorporating these undesirable outputs into the DEA model ensures that more practical, realistic targets are established to improve city performance, leading to a more accurate and actionable efficiency assessment [8].

Furthermore, the role of urban managers in the evaluation process and target setting is crucial. Managerial judgment and practical knowledge can guide the interpretation of model results and the determination of objectives, producing solutions that are both feasible and aligned with the actual conditions of the cities. Without this input, the model may provide only numerical results, and proposed recommendations could be impractical or insufficient. The combination of DEA with undesirable outputs and active managerial involvement provides a practical, applicable framework for improving urban resource management and mitigating pollution, enabling the formulation of optimal policies for cities across Iran.

In this study, the efficiency of Iranian cities in reducing air pollution is evaluated using classical DEA models. The analysis incorporates undesirable outputs, such as CO, NO_2, and PM2.5 concentrations, to reflect each city's environmental performance. This approach allows for a more realistic assessment of urban efficiency by accounting for pollution's negative impacts. The input variables considered in the analysis include green space area and investment in green space development, which are controllable resources available to urban managers. These inputs are used to assess how effectively cities utilize their available resources to mitigate environmental impacts.

First, the efficiency scores for all cities are calculated using DEA models that account for undesirable outputs. This step identifies cities that perform efficiently and those that are relatively inefficient in resource allocation and pollution management. The efficiency score serves as a quantitative measure of each city's performance. Including undesirable outputs in DEA models ensures that higher levels of pollution lower the efficiency score, reflecting the negative consequences of environmental degradation. This adjustment provides a more accurate representation of urban performance than traditional DEA models, which consider only desirable outputs.

Once the efficiency scores are determined, targets for inefficient cities are computed. These targets indicate the adjustments needed in inputs to achieve efficiency without expecting a direct reduction in pollution outputs, which are influenced by multiple uncontrollable factors. This step provides practical guidance for resource allocation.

The targets derived from the DEA highlight the potential improvements that inefficient cities can achieve by optimizing their use of green space and investment resources. By comparing actual inputs with target levels, policymakers can identify specific areas where intervention is necessary. The methodology also allows ranking cities by their efficiency scores. This ranking facilitates benchmarking and comparison, enabling urban planners to learn from more efficient cities and implement best practices in less efficient ones.

The final step involves deriving policy recommendations and management strategies based on the efficiency results and identified targets. These recommendations are tailored to each city's context and focus on maximizing the impact of available resources while acknowledging the limitations imposed by undesirable outputs. This approach provides actionable insights for urban managers, allowing them to prioritize investments in green infrastructure and other controllable inputs. It ensures that resources are allocated effectively to enhance urban environmental performance without unrealistic expectations of immediate pollution reduction [14].

Model (3) is an LP DEA model with Variable Returns to Scale (VRS), specifically designed for situations in which outputs are divided into two distinct types: desirable and undesirable. The main objective of the model is to reduce inputs while keeping the outputs under control. In other words, the model seeks to optimize the decision-making unit's resources without decreasing desirable outputs or increasing undesirable ones.

$$
\begin{aligned}
&\beta_o = \text{Min}\ \beta,\\
&\text{s.t.}\\
&\sum_{j=1}^{n} \mu_j x_{ij} \le \beta x_{io}, i = 1,\dots, m,\\
&\sum_{j=1}^{n} \mu_j y_{rj} = y_{ro}, r = 1,\dots, s1,\\
&\sum_{j=1}^{n} \mu_j y_{rj} \ge y_{ro}, r = 1,\dots, s2,\\
&\sum_{j=1}^{n} \mu_j = 1\\
&\mu_j \ge 0, j = 1,\dots, n.
\end{aligned}
\tag{3}
$$

A key feature of this model is the distinction between output types and the application of different constraints for each. Desirable outputs are indicators the unit aims to maintain or increase, such as production or efficiency. In contrast, undesirable outputs are indicators the unit aims to reduce, such as pollution or waste. The model ensures that desirable outputs remain at their current levels and that undesirable outputs do not exceed their current values, while minimizing inputs to achieve optimal efficiency.

From a managerial perspective, *Model (3)* allows managers to determine the extent to which inputs can be reduced using VRS and an LP approach, while ensuring that outputs remain controlled. This approach enhances resource efficiency while maintaining positive performance and limiting negative impacts, which is especially valuable for organizations and industries with environmental or other adverse side effects.

Overall, the combination of DEA with undesirable outputs offers a robust framework for evaluating urban efficiency. It not only measures performance quantitatively but also supports informed decision-making and strategic planning to improve environmental sustainability in Iranian cities.

4 | Case Study: Evaluating Urban Efficiency in Iranian Cities with Focus on Air Pollution

This case study analyzes the efficiency of 20 major Iranian cities in managing air pollution, using classical DEA models. The analysis considers two input variables: green space area (ha) and investment in green space development (billion Toman), which are key controllable resources for urban managers. The three output variables represent undesirable air pollution indicators: CO (ppm), NO_2 (ppm), and PM2.5 (μg/m³) [15].

Air pollution levels vary significantly across the cities. For instance, Mashhad shows low CO and NO_2 concentrations but extremely high PM2.5 levels (89 μg/m³), while Bandar Abbas has very low CO (0.65 ppm) but high NO_2 (84.59 ppm). These variations highlight the complexity and multidimensionality of urban air pollution management and indicate that resource allocation alone cannot fully control pollution levels.

The first step in the analysis is calculating efficiency scores using DEA models with undesirable outputs. This allows the identification of cities that effectively utilize their green space and investment to mitigate air pollution and those that are inefficient despite significant resource allocation.

Incorporating undesirable outputs ensures that higher pollution levels reduce efficiency scores, reflecting the negative environmental impact. This approach provides a more realistic evaluation than traditional DEA models, which consider only desirable outcomes.

Next, targets for inefficient cities are determined. These targets indicate how cities can adjust inputs—such as expanding green space or optimizing investments—to improve efficiency while acknowledging that external factors beyond immediate control influence some aspects of air pollution.

For example, Karaj, with high green space (6765 ha) and investment (484 billion Toman), may still show lower efficiency due to elevated levels of CO, NO_2, and PM2.5. DEA analysis quantifies the gap between current input allocation and the optimal levels required to manage air pollution better.

The case study also enables benchmarking cities against the most efficient ones. Cities like Tabriz and Urmia, which achieve relatively lower air pollution levels with moderate inputs, serve as models for less efficient cities. Benchmarking provides actionable insights into practical strategies for mitigating urban air pollution.

Policy recommendations are derived from DEA results, efficiency scores, and target inputs, with a specific focus on reducing air pollution. Cities with low efficiency scores are advised to increase green space and optimize investments, while accounting for environmental constraints and urban infrastructure.

This approach emphasizes the critical importance of including undesirable outputs in urban efficiency studies. By explicitly accounting for air pollution indicators, DEA yields practical, achievable targets that guide policymakers toward effective environmental management strategies.

In conclusion, this case study demonstrates that DEA with undesirable outputs, combined with managerial judgment, provides a robust framework for evaluating and improving urban air quality in Iranian cities. The methodology identifies both the strengths and weaknesses of cities, enabling data-driven decision-making for sustainable urban development.

Table 1. Efficiency evaluation of 20 Iranian cities: Green space, investment, and air pollution.

City	Green_Space (ha)	Investment (Billion_Toman)	CO (ppm)	NO_2 (ppm)	PM2.5 (µg/m³)
Tehran	7770	31	3.17	59.2	16
Mashhad	1360	262	0.71	26.64	89
Isfahan	5890	245	3.23	97.26	71.8
Shiraz	5691	354	1.27	79.76	25.9
Tabriz	6234	58	0.79	94.55	10.4
Karaj	6765	484	4.77	90.53	75.2
Qom	966	68	4.85	63.81	66.6
Ahvaz	4926	179	4.14	92.97	68.3
Kermanshah	6078	485	1.87	17.96	71.7
Urmia	8822	197	0.94	27.64	15.9
Rasht	2185	473	3.58	14.07	38.7
Zanjan	1269	280	2.48	39.28	19.3
Sari	7449	199	1.05	44.98	79.1
Arak	2933	455	2.73	34.42	59.9
BandarAbbas	5811	184	0.65	84.59	36.5
Babol	5551	455	4.59	42.11	15.1

Table 1. Continued.

City	Green_Space (ha)	Investment (Billion_Toman)	CO (ppm)	NO_2 (ppm)	PM2.5 (μg/m³)
Yazd	6920	60	1.66	35.28	34.9
Qazvin	1684	373	3.48	58.84	36
Sanandaj	5055	64	1.9	22.68	68.4
Kerman	3885	253	2.84	82.2	61

Based on the data in *Table 1*, the mean and variance of each indicator provide insights into resource distribution and air quality across the 20 cities.

The average green space is approximately 5,445 hectares, but with high variance. Cities such as Urmia, Tehran, and Sari have the most significant green areas, while Qom and Zanjan have very limited green space. This high dispersion indicates a significant imbalance in access to green resources among cities. Investment averages around 253 billion Toman, also with very high variance. Cities like Karaj, Kermanshah, and Arak have the highest investment levels. In contrast, Yazd and Sanandaj have the lowest, reflecting notable differences in cities' financial capacity to develop green spaces and manage pollution.

For air pollution indicators, CO averages about 2.5 ppm, with moderate variance, highest in Karaj and Qom and lowest in Bandar Abbas and Mashhad, indicating the influence of traffic and industrial activities. NO_2 averages approximately 50 ppm with very high variance; the highest levels are observed in Isfahan and Tabriz, and the lowest in Rasht and Kermanshah, reflecting industrial and urban impacts. PM2.5 has an average of around 44 μg/m³, with high variance; Mashhad and Isfahan exhibit very high levels, while Tabriz and Babol show much lower values.

Overall, the data show that both resource allocation (green space and investment) and air quality are highly heterogeneous across Iranian cities. The high variance in these indicators suggests that urban policies should be tailored to each city's conditions, with targeted measures to reduce pollution and optimize resource use.

Table 2. The efficiency scores of the 20 Iranian cities (Model (3)).

DMU	Efficiency
Tehran	0.8739
Mashhad	1.0000
Isfahan	0.3314
Shiraz	1.0000
Tabriz	1.0000
Karaj	1.0000
Qom	0.3645
Ahvaz	0.2829
Kermanshah	1.0000
Urmia	1.0000
Rasht	1.0000
Zanjan	0.8340
Sari	0.8235
Arak	0.8570
BandarAbbas	1.0000
Babol	1.0000
Yazd	0.7246
Qazvin	0.7119
Sanandaj	0.8812
Kerman	0.3942

Based on the data in *Table 2*, the efficiency scores of the 20 Iranian cities can be analyzed as follows:

The average efficiency of cities is approximately 0.80, indicating that most cities perform relatively well in terms of green space, investment, and pollution control. However, the variance is relatively high. Several cities, including Mashhad, Shiraz, Tabriz, Karaj, Kermanshah, Urmia, Rasht, Bandar Abbas, and Babol, achieve full efficiency (1.00), while cities such as Ahvaz (0.28), Isfahan (0.33), and Qom (0.36) exhibit much lower efficiency scores. This dispersion highlights the uneven management of resources and environmental quality across cities. Cities with full efficiency generally have sufficient resources or effective investment and pollution control, whereas cities with lower efficiency may face resource limitations, insufficient investment, or significant environmental challenges. Overall, the efficiency results suggest that although the average performance is high, considerable differences exist among cities. This implies that urban policies should be tailored to each city's conditions, enabling low-performing cities to reach optimal efficiency and achieve targeted improvements in resource utilization and pollution management.

5 | Conclusion

DEA proves to be a valuable quantitative tool for evaluating the efficiency with which cities manage resources and mitigate air pollution. By incorporating both inputs —such as green space and investment —and outputs —including undesirable pollutants like CO_2, NO_2, and PM2.5 —DEA provides a realistic and practical assessment of urban performance. This approach highlights efficiency differences across cities and identifies both high-performing and low-performing urban areas.

The analysis of 20 Iranian cities shows an average efficiency of approximately 0.80, with several cities achieving full efficiency (1.00) and others, such as Ahvaz, Isfahan, and Qom, displaying much lower scores. This dispersion reflects uneven resource management and environmental quality, underscoring the need for policies tailored to each city's specific conditions. Incorporating undesirable outputs ensures that environmental degradation negatively impacts efficiency scores, producing a more accurate representation of urban performance.

DEA-based targets provide actionable guidance for inefficient cities, showing how adjustments in controllable inputs can improve performance without expecting immediate reductions in pollution. By comparing actual resource use with target levels, policymakers can identify priority areas for intervention and benchmark against more efficient cities. The combination of LP DEA models with VRS and managerial judgment creates a practical framework for enhancing urban resource allocation and pollution management, enabling informed policy-making and the implementation of best practices across Iranian cities. For future research, it is recommended to extend this analysis using fuzzy data to account for uncertainties in environmental and pollution indicators. Focus could be placed on large metropolitan areas where resource management challenges and pollution pressures are most significant. Incorporating industrial congestion density as an input or control factor into DEA models can yield a more realistic assessment of efficiency. This approach will help evaluate the direct impact of industrial clusters on undesirable outputs and guide the development of targeted strategies to improve resource use and reduce pollution in major cities.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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References

[1] Lotfi, F. H., Jahanshahloo, G. R., Mozaffari, M. R., & Gerami, J. (2011). Finding DEA-efficient hyperplanes using MOLP efficient faces. *Journal of computational and applied mathematics*, *235*(5), 1227–1231. https://doi.org/10.1016/j.cam.2010.08.007

[2] Kamyab, P., Mozaffari, M. R., Gerami, J., & Wankei, P. F. (2020). Two-stage incentives system for commercial banks based on centralized resource allocation model in DEA-R. *International journal of productivity and performance management*, *70*(2), 427–458. https://doi.org/10.1108/IJPPM-11-2018-0396

[3] Mozaffari, M. R., Gerami, J., Wanke, P. F., Kamyab, P., & Peyvas, M. (2022). Ratio-based data envelopment analysis: An interactive approach to identify benchmark. *Results in control and optimization*, *6*, 100081. https://doi.org/10.1016/j.rico.2021.100081

[4] Ostovan, S., Mozaffari, M. R., Jamshidi, A., & Gerami, J. (2020). Evaluation of two-stage networks based on average efficiency using DEA and DEA-R with fuzzy data. *International journal of fuzzy systems*, *22*(5), 1665–1678. https://doi.org/10.1007/s40815-020-00896-9

[5] Gerami, J., Mozaffari, M. R., Wanke, P. F., & Correa, H. L. (2022). Improving information reliability of non-radial value efficiency analysis: An additive slacks based measure approach. *European journal of operational research*, *298*(3), 967–978. https://doi.org/10.1016/j.ejor.2021.07.036

[6] Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research*, *2*(6), 429–444. https://doi.org/10.1016/0377-2217(78)90138-8

[7] Nejati, M., Rostamy-Malkhalifeh, M., Kargar, R., & Mehrjoo, M. N. (2018). The most efficient unit in data envelopment analysis using genetic algorithm. *Theory of approximation and applications*, *12*(2), 73-81. (**In Persian**). https://journals.iau.ir/article_546138.html

[8] Mozaffari, M. R., Mohammadi, S., Wanke, P. F., & Correa, H. L. (2021). Towards greener petrochemical production: Two-stage network data envelopment analysis in a fully fuzzy environment in the presence of undesirable outputs. *Expert systems with applications*, *164*, 113903. https://doi.org/10.1016/j.eswa.2020.113903

[9] Gerami, J., Reza Mozaffari, M., & Wanke, P. F. (2020). A multi-criteria ratio-based approach for two-stage data envelopment analysis. *Expert systems with applications*, *158*, 113508. https://doi.org/10.1016/j.eswa.2020.113508

[10] Sohrabi, A., Gerami, J., & Mozaffari, M. R. (2022). A novel inverse DEA-R model for inputs/output estimation. *Journal of mathematical extension (JME)*, *16*(8). https://www.ijmex.proux.ir/index.php/ijmex/article/view/2047

[11] Kao, C., & Hung, H.-T. (2005). Data envelopment analysis with common weights: The compromise solution approach. *Journal of the operational research society*, *56*(10), 1196–1203. https://doi.org/10.1057/palgrave.jors.2601924

[12] Olfati, M., Krömer, P., Fanati Rashidi, S., Mirjalili, S., & Snášel, V. (2025). A goal programming-based algorithm for solving multi objective optimization problems. *Annals of operations research*. https://doi.org/10.1007/s10479-025-06646-0

[13] Wanke, P., Ostovan, S., Mozaffari, M. R., Gerami, J., & Tan, Y. (2022). Stochastic network DEA-R models for two-stage systems. *Journal of modelling in management*, *18*(3), 842–875. https://doi.org/10.1108/JM2-10-2021-0256

[14] Despotis, D. K. (2002). Improving the discriminating power of dea: Focus on globally efficient units. *Journal of the operational research society*, *53*(3), 314–323. https://doi.org/10.1057/palgrave.jors.2601253

[15] Hosseinzadeh Lotfi, F., Noora, A. A., Jahanshahloo, G. R., Jablonsky, J., Mozaffari, M. R., & Gerami, J. (2009). An MOLP based procedure for finding efficient units in DEA models. *Central european journal of operations research*, *17*(1), 1–11. https://doi.org/10.1007/s10100-008-0071-1