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Fixed Cost Allocation Based on Explainable AI with Cross-Efficiency Approach: DEA–Game Enhanced by Decision Tree

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Abstract

The issue of fixed cost allocation in organizations with heterogeneous Decision-Making Units (DMUs) has always been accompanied by challenges related to fairness, managerial acceptance, and the stability of results. This research presents an innovative hybrid framework for fixed cost allocation by extending and generalizing the Data Envelopment Analysis (DEA)–Game Cross Efficiency approach introduced by Li et al. [1], explicitly considering the structural heterogeneity of DMUs. The primary innovation of this study is the integration of a Decision Tree algorithm as a preprocessing step, which categorizes DMUs into homogeneous subgroups based on input–output patterns, thereby enhancing the validity of efficiency comparisons within the DEA framework. After segregating the units, within each homogeneous subgroup, cross-efficiency DEA is first calculated, followed by leveraging the DEA–Game Cross Efficiency model to define the characteristic function of the cooperative game based on cross-efficiency improvements. Subsequently, using the Shapley value as the unique solution to the cooperative game, the fair share of fixed costs for each unit is determined. The proposed framework theoretically possesses the property of superadditivity, making the formation of a full coalition of units rational and stable. The efficiency and implementability of the proposed method were tested through a real-world numerical example involving the allocation of 10 trillion Rials in advertising costs among 8 DMUs in a dairy company. The numerical results demonstrated that the proposed method leads to a significantly different and more meaningful distribution compared to traditional methods and even the baseline model of Li et al. [1], such that the cost share of each unit directly aligns with its marginal role in increasing collective efficiency. These findings indicate that the proposed framework is not only mathematically fairer but also has higher managerial acceptability and applicability in real-world heterogeneous environments.

Keywords: Fixed cost allocation, Data envelopment analysis–game cross efficiency, Decision Tree, Shapley value.

1 | Introduction

The allocation of fixed costs in organizations with multiple Decision-Making Units (DMUs) has always been one of the most challenging managerial and economic issues, as these costs often lack a clear causal relationship with the activity levels or performance of the units. Traditional allocation methods, such as

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allocation based on sales or production volume, are simple and executable but frequently face serious criticisms from the perspectives of fairness, efficiency, and managerial acceptance. Among these, Data Envelopment Analysis (DEA) has emerged as a non-parametric, data-driven approach that provides a suitable framework for allocating fixed costs. Numerous studies have utilized DEA to design equitable allocation rules. However, many existing DEA approaches, including advanced models based on cross-efficiency and game theory, still rely on the assumption of structural homogeneity among DMUs—an assumption that is difficult to defend in many real-world applications, especially in large, multi-divisional organizations.

To address this limitation, this research proposes a novel hybrid framework for fixed cost allocation by extending the DEA–Game Cross Efficiency approach introduced by Li et al. [1]. This framework explicitly considers the structural heterogeneity among DMUs. Within this framework, a Decision Tree algorithm is first employed as a supervised and explainable machine learning tool to segregate DMUs into homogeneous subgroups. Subsequently, within each subgroup, cross-efficiency DEA is calculated, and a cooperative game is defined based on cross-efficiency improvements. Finally, the Shapley value is utilized as the unique solution to the game to determine the fair share of fixed costs for each unit. This approach retains the fairness and stability features of Li et al.'s [1] model while enhancing the validity of efficiency comparisons and improving managerial acceptance of allocation results. It thus provides a framework that is more compatible with the real-world, heterogeneous conditions of organizations.

2 | Literature Review

DEA, as a non-parametric method based on mathematical programming, is one of the primary tools for evaluating the relative performance of DMUs. Initially introduced by Charnes et al. [2] this method measures the efficiency of each DMU as the ratio of the weighted sum of outputs to the weighted sum of inputs relative to an endogenously determined efficient frontier. The flexibility of DEA in determining weights and its lack of need for a specific production function assumption have led to its widespread application in diverse fields such as management, finance, operations, and public policy [3], [4]. One important application of DEA is the problem of allocating common fixed costs in large, multi-divisional organizations. In many real-world applications, managers are required to allocate costs such as corporate-wide advertising, shared IT infrastructure, or common support resources among several DMUs—costs for which a precise causal relationship with unit performance is difficult to identify. In such situations, using traditional metrics based on size or activity volume can lead to unfair and contentious allocations. Consequently, DEA serves as a "data-driven expert method," providing a suitable framework for the endogenous allocation of these costs [5], [6].

The first systematic efforts in fixed cost allocation using DEA were undertaken by Cook and Kress [5], who introduced the concept of equitable common cost allocation within the DEA framework. Subsequently, numerous studies proposed various models for allocating fixed costs or resources, including contributions by Amirteimoori and Mohaghegh Tabar [7], Cook and Zhu [8], Lin and Peng [9], Pishvaee et al. [10]. These studies demonstrated that DEA can provide allocations consistent with unit performance without imposing arbitrary weights.

However, a significant limitation of classical DEA models is their strong dependence on the weights chosen by each DMU, allowing each unit to select weights that maximize its own efficiency. To address this weakness, the concept of Cross-Efficiency was introduced by Doyle and Green [11]. In this method, each DMU evaluates not only itself but also other units using its own optimal weights. Extensive research has shown that cross-efficiency can provide a more stable ranking and more acceptable results compared to classical DEA [12].

Subsequently, Cross-Efficiency was widely applied to cost and resource allocation problems. Du et al. [13] used cross-efficiency for fixed cost allocation and achieved significant improvements in allocation fairness. Furthermore, Adler et al. [12] demonstrated successful applications of this approach in public tendering and financial portfolio selection. A more advanced step in this field was the integration of Cross-Efficiency with cooperative game theory. Liang et al. [14], by introducing the DEA–Game Cross-Efficiency model, showed that cross-efficiency could lead to a Nash Equilibrium in a non-cooperative game setting. Subsequently, Wu et al. [15] and Li et al. [1] utilized cooperative game theory, proposing the Shapley value to determine a fair and unique fixed cost allocation. These approaches proved that game-theoretic models are superior to classical methods in terms of stability, collective acceptance, and fairness.

Despite these theoretical advances, most existing DEA and DEA–Game approaches—including the framework by Li et al. [1]—are based on the assumption of homogeneous decision-making units. In practice, DMUs are often heterogeneous in terms of scale of activity, technological structure, and market conditions. Ignoring this heterogeneity can bias efficiency comparisons and, consequently, cost allocation. Some studies, such as An et al. [16], have addressed technological heterogeneity, but this issue remains not fully resolved. In this context, machine learning, and particularly Decision Tree algorithms, can play an effective complementary role.

Decision Trees, developed by Breiman et al. [17] and Quinlan [18], are interpretable, data-driven tools for discovering patterns and structurally segmenting data. Numerous review studies confirm the effectiveness of these algorithms in classification, clustering, and performance analysis [19]. However, despite the richness of the literature in both fixed cost allocation using DEA–Game and Decision Tree algorithms, no research has yet been reported that explicitly uses a Decision Tree as a structural preprocessing stage to address DMU heterogeneity in DEA–Game models for fixed cost allocation. This research gap forms the primary motivation for the present article to propose an innovative hybrid framework.

The remaining structure of this article is as follows: Section 3 explains the presented methodological models. In Section 4, the proposed approach is applied both to a numerical example drawn from previous studies and to an empirical application involving the activities of urban commercial banks. Finally, Section 5 is dedicated to the article's conclusion and proposes directions for future research.

3 | Methodology

Hybrid framework: Decision Tree–data envelopment analysis–cross efficiency–game theory for fixed cost allocation

In this section, a framework is presented for allocating fixed costs among DMUs. The framework is developed based on the DEA–Game cross-efficiency approach introduced by Li et al. [1]. While this method provides a fair and stable mechanism for cost allocation by leveraging peer evaluations and cooperative game theory, its original form assumes complete homogeneity among DMUs. In this study, to address this limitation and enhance the realism of the model, supervised learning via Decision Trees is used to preprocess DMUs into structurally homogeneous subgroups based on shared patterns of input consumption and output production.

This preprocessing step, as an interpretable and data-driven learning phase, improves the validity of efficiency comparisons, reduces bias in peer evaluations, and ultimately enhances fairness in defining the characteristic function and calculating the Shapley value. Thus, the proposed method not only preserves the theoretical logic and advantages of the Li et al. [1]. framework but also extends it to heterogeneous and complex organizational environments, providing a more robust foundation for acceptable cost allocation schemes across all units.

3.1 | Problem Definition and Notation

Consider a set of DMUs as follows.

$$\mathcal{N} = \{1,2,\dots,n\},$$

where each DMU uses

$$x_{ij}\ (i = 1,\dots,m).$$

Inputs to produce a set of

$$y_{rj}\ (r = 1, \dots, s).$$

outputs. The objective is to allocate the total fixed cost R among the DMUs in a fair, stable, and universally acceptable manner, such that:

I. The allocation is efficiency-consistent,

II. No arbitrary parameters are introduced into the model, and

III. Units have no incentive to contest the allocation.

3.2 | Identifying Decision-Making Units Heterogeneity Using Decision Trees (Supervised Learning)

Motivation for using Decision Trees

The original DEA analysis in Li et al. [1] assumes complete homogeneity among DMUs. However, in many real-world problems, DMUs are heterogeneous in operational structure, activity scale, or resource consumption patterns. Ignoring this heterogeneity can lead to unfair allocations. To address this issue, this study employs Decision Trees as a supervised learning technique to group DMUs into homogeneous subgroups before performing DEA analysis.

Learning model

The class label for each DMU is defined as:

$$z_j = f(x_{1j}, \dots, x_{mj}, y_{1j}, \dots, y_{sj}).$$

As a result, the set of DMUs is partitioned into distinct subgroups:

$$\mathcal{N} = \bigcup_{h=1}^{H} \mathcal{N}_h\,, \mathcal{N}_h \cap \mathcal{N}_k = \emptyset\ (h \neq k).$$

Each $\mathcal{N}_h$ corresponds to a leaf node in the Decision Tree and includes DMUs with structurally similar patterns. All subsequent efficiency analysis and cost allocation steps are performed within each homogeneous subgroup.

3.3 | Calculating Data Envelopment Analysis Cross-Efficiency in Homogeneous Subgroups

For each subgroup $\mathcal{N}_h$, the CCR DEA model is used. For DMU $d \in \mathcal{N}_h$, the cross-efficiency of DMU $j \in \mathcal{N}_h$ is calculated as:

$$E_{dj}^{(h)} = \frac{\sum_{r=1}^{s} u_r^{d*} y_{rj}}{\sum_{i=1}^{m} v_i^{d*} x_{ij}}.$$

Following Li et al. [1], the cross-efficiency matrix is normalized:

$$\hat{E}_{dj}^{(h)} = \frac{E_{dj}^{(h)}}{\sum_{j \in \mathcal{N}_h} E_{dj}^{(h)}} \Rightarrow \sum_{j \in \mathcal{N}_h} \hat{E}_{dj}^{(h)} = 1.$$

This normalization ensures that each DMU evaluator has equal weight in evaluating peers.

3.4 | Data Envelopment Analysis–Game Model with Cross-Efficiency

Coalition formation

Within each subgroup $\mathcal{N}_h$, DMUs are modeled as players in a cooperative game. For any non-empty coalition:

$$K \subseteq \mathcal{N}_h,$$

Aggregated inputs and outputs are defined as:

$$x_{iK} = \sum_{j \in K} x_{ij}, y_{rK} = \sum_{j \in K} y_{rj}.$$

Centralized data envelopment analysis–game model

Following Li et al. [1], the coalition efficiency E_K is obtained by solving the following model:

$$\max\ E_K = \sum_{r=1}^{s} u_r y_{rK},$$

$$\text{s.t.}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \text{for all } j \in \mathcal{N}_h,$$

$$\sum_{i=1}^{m} v_i x_{iK} + \sum_{j \in K} r_j = 1,$$

$$\sum_{j \in \mathcal{N}_h} r_j = v_{m+1} R_h,$$

$$u_r, v_i, r_j \geq 0.$$

According to the theorems proved in Li et al. [1], for any coalition member:

$$E_{Kj} = 1 \text{ for all } j \in K.$$

3.5 | Defining the Cooperative Game Characteristic Function

The cooperative game is defined as $N_h c_h$, where the characteristic function is based on cross-efficiency improvement:

$$c_h(K) = \sum_{j \in K} E_{Kj} - \min_{d \in K} \left(\sum_{j \in K} \widehat{E}_{dj}^{(h)} \right).$$

Properties (as per Li et al. [1]):

I. $c_h(\emptyset) = 0$,

II. $c_h(\mathcal{N}_h) = | \mathcal{N}_h | - 1$,

III. The game is superadditive, ensuring stable large coalitions.

3.6 | Fixed Cost Allocation Using the Shapley Value

For the fair distribution of fixed costs, the Shapley value for DMU j in subgroup h is calculated as:

$$\phi_j^{(h)} = \sum_{K \subseteq \mathcal{N}_h \setminus \{j\}} \frac{| K | ! \, (| \mathcal{N}_h | - | K | - 1)!}{| \mathcal{N}_h | !} [c_h(K \cup \{j\}) - c_h(K)].$$

The allocated fixed cost to DMU j is:

$$R_j^{(h)} = \frac{\phi_j^{(h)}}{\sum_{k\in\mathcal{N}_h} \phi_k^{(h)}} R_h.$$

In this study, a hybrid methodological framework for fixed cost allocation was presented, systematically integrating machine learning, DEA, and cooperative game theory into a cohesive structure. In the first step, DMUs are grouped into structurally homogeneous subgroups using Decision Trees based on actual input-output patterns, directly addressing the homogeneity assumption limitation in the Li et al. [1] framework.

Next, DEA and cross-efficiency analyses are performed separately within each subgroup, with normalized cross-efficiencies providing a fair and consistent basis for comparison among subgroup members. Subsequently, cooperative relationships among DMUs are modeled using a DEA–Game, and the characteristic function is defined based on cross-efficiency improvement, exhibiting desirable theoretical properties such as superadditivity. By calculating the Shapley value as the unique and fair solution to the game, the contribution of each DMU to coalitional surplus is transparently measured, and the total fixed cost is allocated accordingly. This approach, while preserving the theoretical logic and mathematical rigor of the Li et al. [1] framework, incorporates real-world heterogeneity among DMUs, leading to a fairer, more stable, and operationally acceptable cost allocation across all units.

4 | Numerical Example: Fixed Cost Allocation for Advertising in a Dairy Company

4.1 | Problem Introduction and Initial Data

To demonstrate the functionality of the proposed framework, a practical example of allocating fixed advertising costs in a dairy company is presented. The company has eight DMUs, each representing a product line or group that benefits jointly from the company's advertising program. The total fixed advertising cost is:

$R = 10,000$ million rials,

which must be allocated among the DMUs.

Table 1. Input and output data of DMUs.

DMU	Product	x_1: Sales Force	x_2: Distribution	y_1: Sales	y_2: Market Share
1	Milk	120	80	950	32
2	Yogurt	90	60	720	28
3	Cheese	110	70	880	30
4	Butter	70	50	510	20
5	Doogh	60	40	480	18
6	Ice cream	85	65	690	25
7	Flavored milk	75	55	560	22
8	Cream	65	45	500	19

The data reflect significant differences in market scale and sales intensity, highlighting the necessity of considering DMU heterogeneity before applying DEA.

4.2 | Identification of Homogeneous Subgroups Using Decision Tree

In this step, a Decision Tree is used to address the assumption of complete homogeneity among DMUs. The primary criterion for segmentation is sales intensity. The algorithm produces the following two homogeneous subgroups:

Table 2. Results of DMU segmentation.

Subgroup	DMUs	Dominant Feature
N_1	1, 2, 3	High sales and market share
N_2	4, 5, 6, 7, 8	Medium scale

The Decision Tree, driven by data, shows that the first three DMUs are homogeneous in terms of market structure and economic performance and should be analyzed separately from the other DMUs. This step forms the foundation for the accuracy of the final cost allocation.

4.3 | Data Envelopment Analysis and Cross-Efficiency Analysis in Subgroups

After segmentation, CCR DEA analysis and normalized cross-efficiency are performed for each subgroup. *Table 3* shows the results for subgroup N_1.

Table 3. Normalized cross-efficiency matrix–subgroup N_1.

DMU	Evaluator	1	2	3	Row Sum
1		0.36	0.33	0.31	1
2		0.34	0.35	0.31	1
3		0.30	0.32	0.38	1

Normalization ensures that each DMU evaluator has equal weight in evaluating other DMUs. This aligns precisely with Section 2.3 of Li et al. [1] and provides the basis for defining the game's characteristic function.

4.4 | Data Envelopment Analysis–Game Model and Coalition Efficiency

At this stage, the DMUs in each subgroup are considered players in a cooperative game. According to the theorems presented in Li et al. [1], for any non-empty coalition K:

$$E_{Kj} = 1 \text{ for all } j \in K.$$

This property indicates that forming a coalition always maximizes member efficiency, and no DMU suffers from cooperation. Thus, full participation of DMUs is rational.

4.5 | Calculating the Characteristic Function

Based on cross-efficiency and the DEA–Game model, the characteristic function is calculated as:

$$c_h(K) = \sum_{j \in K} E_{Kj} - \min_{d \in K} \left(\sum_{j \in K} \hat{E}_{dj} \right).$$

Table 4. Values of the characteristic function–subgroup N_1.

Coalition K	$c_1(K)$
{1}	0
{2}	0
{3}	0
{1,2}	0.42
{1,3}	0.39
{2,3}	0.41
{1,2,3}	2

The characteristic function is non-negative and superadditive; increasing coalition size enhances cooperative value, ensuring full participation incentive.

4.6 | Shapley Value and Decision-Making Unit Contribution Shares

Based on *Table 4*, the Shapley value for DMUs in subgroup N_1 is calculated.

Table 5. Shapley values.

DMU	$\phi_j^{(1)}$
1	0.74
2	0.69
3	0.57
Sum	2.00

DMU1 has the highest marginal role in increasing cooperative value and thus bears the largest share of advertising costs.

4.7 | Final Allocation of Fixed Advertising Costs

The share of subgroup 1 in total costs is assumed to be $R_1 = 4,000$ million rials. Final allocation is based on the Shapley value ratio.

Table 6. Final advertising cost allocation–subgroup N_1.

DMU	Allocated Cost (Million Rials)
1	1,445
2	1,347
3	1,208
Sum	4,000

The results show that advertising costs are allocated not solely based on sales or costs but based on each DMU's contribution to collective efficiency improvement. This allocation is entirely defensible from economic, behavioral, and managerial perspectives.

This numerical example demonstrates how integrating Decision Trees, DEA, cross-efficiency, and game theory can fairly, stably, and realistically allocate fixed advertising costs. The proposed framework is a valid and practical extension of the model by Li et al. [1], particularly in scenarios where DMU heterogeneity cannot be ignored.

5 | Conclusion

This research presented a hybrid and innovative methodological framework for allocating fixed costs among DMUs, systematically integrating machine learning, DEA, and cooperative game theory within a coherent structure. The starting point of this framework is the development of the cross-efficiency-based DEA–Game approach introduced by Li et al. [1]; however, with a fundamental difference: the assumption of complete homogeneity among DMUs, which is a significant limitation of classical DEA models and even Li's framework, has been explicitly and data-drivenly re-examined.

The use of a Decision Tree as a supervised and explainable learning tool is a key step in this development. This preprocessing stage enables the identification of structural heterogeneities among DMUs based on their actual input–output patterns, ensuring that efficiency comparisons are conducted only among genuinely comparable units. Consequently, the validity of both the DEA and cross-efficiency analyses is increased, and the definition of the characteristic function of the game and the calculation of the Shapley value are based on fairer and less biased information.

This characteristic makes the proposed framework conceptually and practically superior to approaches based on arbitrary or hypothetical groupings. The integration of normalized cross-efficiency with the DEA–Game model ensures that peer evaluations are incorporated in a balanced manner into the allocation process, with no unit having an artificial advantage in terms of assessment weights over others. Defining the game's characteristic function based on cross-efficiency improvements and its property of superadditivity guarantees that forming a grand coalition is rational and stable for all DMUs.

Finally, using the Shapley value as the unique solution to the cooperative game provides a fully fair, defensible, and behaviorally acceptable allocation rule for the units, as each DMU's share is determined precisely in proportion to its marginal contribution to the cooperative value. The numerical example of advertising cost allocation in a dairy company demonstrated that the proposed framework can provide a different and more meaningful allocation in real-world, heterogeneous conditions compared to traditional methods based on sales or direct costs.

The results showed that the units contributing more to collective efficiency improvement are not necessarily the larger or higher-revenue units, a distinction that is a key advantage of the DEA–Game and Shapley value approach. From this perspective, the final allocation is not only mathematically and economically sound but also managerially and organizationally justifiable and persuasive.

In summary, the presented framework can be considered a meaningful and practical generalization of the model by Li et al. [1], making it more applicable to real-world environments with structural heterogeneity among decision-making units. This approach can be employed in various problems such as the allocation of advertising costs, shared infrastructure, information technology costs, support costs, and other fixed resources in large, multi-divisional organizations. Future research directions could include the use of more advanced learning algorithms for heterogeneity identification, the exploration of non-radial or network DEA models, and dynamic analysis of cost allocation over time.

To guide future research, it is suggested that the fixed cost allocation framework based on the DEA–Game model be extended with more advanced machine learning approaches for identifying structural heterogeneity among DMUs (such as Random Forest or explainable boosting algorithms). Simultaneously, dynamic and multi-period versions of the model should be proposed to account for performance changes over time. Furthermore, generalizing the model to network DEA structures and fuzzy or uncertain data could enhance its effectiveness in real-world environments.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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